*June* 18, 1896—*continued.*

VI. "Experimental Proof of van't Hoff's Constant, Dalton's Law, &c., in very dilute Solutions." By Dr. MEYER WILDERMANN. Communicated by Professor FITZGERALD, F.R.S.

VII. "On the Determination of the Wave-length of Electric Radiation by Diffraction Gratings." By J. C. BOSE. Communicated by LORD RAYLEIGH, Sec. R.S.

VIII. "The Effects of a strong Magnetic Field upon Electric Discharges in Vacuo." By A. A. C. SWINTON. Communicated by LORD KELVIN, F.R.S.

IX. "On the Structure of Metals, its Origin and Changes." By M. F. OSMOND and Professor ROBERTS-AUSTEN, C.B., F.R.S.

X. "Magnetisation of Liquids." By JOHN S. TOWNSEND. Communicated by Professor J. J. THOMSON, F.R.S.

XI. "Selective Absorption of Röntgen Rays." By J. A. MCCLELLAND. Communicated by Professor J. J. THOMSON, F.R.S.

XII. "On the Determination of Freezing Points." By J. A. HARKER, D.Sc. Communicated by Professor SCHUSTER, F.R.S.

XIII. "The Menstruation and Ovulation of *Macacus rhesus*; with Observations on the Changes undergone by the discharged Follicle. Part II." By WALTER HEAPE. Communicated by Dr. M. FOSTER, Sec. R.S.

XIV. "Phenomena resulting from Interruption of Afferent and Efferent Tracts of the Cerebellum." By Dr. J. S. RISIEN RUSSELL. Communicated by Professor V. HORSLEY, F.R.S.

The Society adjourned over the Long Vacation to Thursday, November 19.

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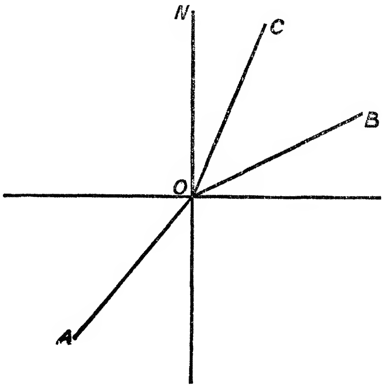
## "Angular Measurement of Optic Axial Emergences." By WILLIAM JACKSON POPE. Communicated by Professor ARMSTRONG, F.R.S. Received February 7,—Read March 19, 1896.

Crystals belonging to the monoclinic or anorthic systems are rarely obtained in which the optical orientation is such that a large crystal face is so nearly perpendicular to a bisectrix that the apparent optic axial angle as observed in air can be directly measured by means of the ordinary Fuess apparatus. It thus becomes necessary to first grind plates of known orientation for optical examination; this latter operation is by no means easily performed, especially in the case of brittle organic substances. Very usually, however, crystals belonging to the biaxial systems are obtained in which an optic axis apparently emerges into air through a particular face; in these cases the accurate measurement of the angle between the apparent direction in air of the optic axis and the normal to the crystal plate becomes an important element in the determination of the optical constants of the crystal.

The ordinary method of determining this angle is a direct one; the crystal is adjusted in the optic axial angle apparatus and a reading is taken for the above emergence, after the position of the normal to the plate has been found by reflecting a beam of light down the telescope tube and turning the crystal until the shadow and reflected image of the crosswires coincide; the angular difference between the two readings is then the required apparent angle of emergence into air. This method of finding the position of the normal is, however, very tedious, and, unless the crystal plate possesses a highly polished surface, very inaccurate.

To remedy these defects a method has been devised of indirectly determining this angle by calculating it from the angle through which the optic axis is apparently refracted by an oil of high refractive index. The crystal is mounted and adjusted in the optic axial angle apparatus in the ordinary way, and a reading is taken for the optic axial emergence in air; a parallel-sided glass cell containing α-bromonaphthalene or some other highly refractive liquid is then raised until it surrounds the crystal, and a second reading is taken of the apparent emergence of the optic axis. From the difference between these two angular readings the angle of emergence into air can be calculated, if the index of refraction of the oil is known.



In the figure, OA is an optic axial direction in the crystal, OB is the direction of optic axial emergence into air, and OC is the direction of emergence into a liquid of refractive index $\mu$; ON is the normal to the crystal plate. Then $\alpha$, the angle of emergence into air, is NOB, whilst $\theta$, the angle of emergence into the liquid is NOC and $\sin\alpha / \sin\theta = \mu$; it is required to calculate the angle $\alpha$, from the observed value of $\alpha - \theta$.

Then, since $\sin\alpha/\sin\theta = \mu$,

$$\frac{1}{\mu} = \frac{\sin\{\alpha-(\alpha-\theta)\}}{\sin\alpha}$$

$$= \frac{\sin\alpha\cos(\alpha-\theta)-\cos\alpha\sin(\alpha-\theta)}{\sin\alpha}$$

$$= \cos(\alpha-\theta)-\cot\alpha\sin(\alpha-\theta)$$

and

$$\cot\alpha = \cot(\alpha-\theta)-\frac{1}{\mu\sin(\alpha-\theta)} \quad\ldots\ldots\ldots\ldots \quad (1)$$

Or again, since $\sin\alpha/\sin\theta = \mu$,

$$\frac{\mu+1}{\mu-1} = \frac{\sin\alpha+\sin\theta}{\sin\alpha-\sin\theta}$$

$$= \frac{\sin\frac{1}{2}(\alpha+\theta)\cos\frac{1}{2}(\alpha-\theta)}{\sin\frac{1}{2}(\alpha-\theta)\cos\frac{1}{2}(\alpha+\theta)},$$

whence

$$\tan\frac{\alpha+\theta}{2} = \frac{\mu+1}{\mu-1}\tan\frac{\alpha-\theta}{2} \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

a form more convenient than (1) for logarithmic calculation.

To test the accuracy of the method, measurements have been made on biaxial plates of different optical properties, liquids of various refractive indices being used. The index of refraction of the liquid employed is conveniently determined with the Pulfrich refractometer; the refraction is so affected by differences of temperature and of purity that it is necessary to determine it for the liquid as actually used; the liquid does not need to be specially purified. The measurements given in the two appended tables were made on plates of topaz, each of them cut perpendicularly to the acute bisectrix. By measurement of the optic axial angles, the apparent emergences into air for sodium light were found to be 53° 24′ and 54° 42′, respectively.

These two sets of measurements suffice to show that the method possesses very considerable accuracy, although the values of $\alpha-\theta$ measured are not very large; the numbers also seem to indicate

Table I.—Plate with angle of emergence in air 53° 24′.

| Liquid. | $\mu_D$. | $a-\theta$. | $a$. | Δ. |
|---|---|---|---|---|
| Carbon bisulphide...... | 1·6473 | 24° 15′ | 53° 26′ | +2′ |
| $a$-Bromonaphthalene.... | 1·5341 | 21 50 | 53 22½ | −1½ |
| Benzene . ............ | 1·4970 | 21 0 | 53 27½ | +3½ |
| Turpentine............ | 1·4726 | 20 20 | 53 20½ | −3½ |
| Olive oil.............. | 1·4673 | 20 12 | 53 21 | −3 |
| Glycerol.............. | 1·4634 | 20 10 | 53 28½ | +4½ |
| Chloroform ........... | 1·4439 | 19 35 | 53 19½ | −4½ |
| Alcohol.......... .... | 1·3561 | 17 2 | 53 15 | −9 |
| Water.... ........... | 1·3327 | 16 21 | 53 22 | −2 |

Table II.—Plate with angle of emergence in air 54° 42′.

| Liquid. | $\mu_D$. | $a-\theta$. | $a$. | Δ. |
|---|---|---|---|---|
| Carbon bisulphide...... | 1·6473 | 24° 58′ | 54° 38½′ | −3½′. |
| $a$-Bromonaphthalene.... | 1·5341 | 22 35 | 54 44½ | +2½ |
| Benzene............... | 1·4970 | 21 41 | 54 44½ | +2½ |
| Turpentine............ | 1·4726 | 21 0 | 54 37 | −5 |
| Olive oil.. ........... | 1·4673 | 20 56 | 54 45 | +3 |
| Glycerol.............. | 1·4634 | 20 51 | 54 47½ | +5½ |
| Chloroform............ | 1·4439 | 20 17 | 54 42 | 0 |
| Alcohol............... | 1·3561 | 17 45 | 54 48½ | +6½ |
| Water................ | 1·3327 | 16 54 | 54 37 | −5 |

that, as would of course be expected, the most accurate results are obtained with liquids of high refractive index, which give comparatively large values of $a-\theta$. By determining the values of $a-\theta$ for each of two optic axes of a given crystal plate, it can easily be ascertained with what amount of accuracy the plate has been cut perpendicularly to the bisectrix.

The principle of the method here described may very possibly be advantageously employed in other branches of optical investigation.

### “On Colour Photography by the Interferential Method.” By G. Lippmann, Professor of Physics, Faculty of Sciences, Paris. Communicated by Sir Joseph Lister, Bart., P.R.S.

Received April 14,—Read April 23, 1896.

Colour photographs of the spectrum, or of any other object, are obtained by the following method. A *transparent* photographic film of any kind has to be placed in contact with a metallic *mirror* during